Tensionless branes and discrete gauge symmetry

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We argue that the tensionless branes recently found on non-BPS D-branes using noncommutative field theory are, in fact, gauge equivalent to the vacuum under a discrete gauge symmetry. We also give a simple construction of the D$(2p)$-branes in type IIA theory starting from a single non-BPS D9-brane.

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It has been recently shown that D-branes can be constructed as exact solitons in open string field theory using techniques of noncommutative field theory [1,2,3,4]. The resulting solutions have the right tension and spectrum to be identified with D-branes [3]. For superstrings, one can construct Bogomol'nyi-Prasad-Sommerfield (BPS) Dp-branes, as well as non-BPS D-branes, as noncommutative solitons. All these solitons can be analyzed directly in open string field theory [4].

These results agree precisely with expectations, but there is an additional surprise: in type II string theory, there are also tensionless p-branes [2,3]. If these were genuine light states in type II string theory, they should have already been known from other studies. In this paper we argue that the tensionless solutions are actually gauge transformations of the vacuum, so they do not appear as new states in the physical spectrum. We use this observation to give a new construction of D$(2p)$-branes starting from an unstable non-BPS D9-brane of type IIA theory.

I. THE BRANE-ANTIBRANE SYSTEM

Although the emphasis in [2,3] was on solitons on unstable Dp-branes, it is useful to first consider the D$p-\overline{\mathrm{D}p}$ system. We start by reviewing this system in the absence of a background B field and for concreteness we start in type IIB theory. This system has two gauge fields A_+, A_-, and a complex tachyon field T transforming with a charge $(1,-1)$ under $U(1)_+\otimes U(1)_-$. The gauge transformation laws are

$$T\rightarrow UTW^\dagger,$$
$$\mathrm{D}_+\rightarrow U\mathrm{D}_+U^\dagger,$$
$$\mathrm{D}_-\rightarrow W\mathrm{D}_-W^\dagger, \qquad (1.1)$$

with $U\in U(1)_+$, $W\in U(1)_-$, and $\mathrm{D}_\pm=d+A_\pm$.

We now consider, as in [2,3], turning on a background B field and taking the limit of large noncommutativity. For concreteness we consider a B field in two directions, $B_{89}=B$, and take $p=9$; the constructions below are easily generalized to more general B fields. In this limit we can drop ordinary derivatives in the noncommutative directions, T becomes an arbitrary complex operator on Hilbert space $\mathcal{H}$, and U, W are independent unitary transformations on $\mathcal{H}$. The equations of motion become $V'(T)=0$, where operator multiplication is implied [1].

We take the classical tachyon potential V to have a local maximum at $T=0$ and a ring of minima at $|T|=t_*$. According to the conjecture of Sen [5], $T=0$ represents the unstable D9$-\overline{\mathrm{D9}}$ system and $|T|=t_*$ represents the closed-string vacuum with no open string excitations. The solution in the noncommutative theory [2,3]

$$T=-t_*(1-P_k), \qquad (1.2)$$

with P_k a rank k projection operator on $\mathcal{H}$, represents k BPS D7-branes, while the tensionless-brane solutions are given by

$$T=-t_*(1-2P_k). \qquad (1.3)$$

Choosing a basis of $\mathcal{H}$ with a diagonal P_k, we can write the tensionless brane solution as $T=\mathrm{diag}(t_*^k,-t_*,-t_*,\ldots)$, where the power on t_* denotes repeated entries, so there are k entries $+t_*$. This is clearly gauge equivalent to the vacuum configuration $T=-t_*1$ using the gauge transformation $U=\mathrm{diag}(-1^k,1,1,\ldots), W=1$.

II. THE NON-BPS BRANE

We now turn our attention to the unstable Dp-branes. One can obtain a non-BPS Dp-brane from a D$p-\overline{\mathrm{D}p}$ system by projecting with respect to $(-1)^{F_L}$ [6]. This projection sets $A_+=A_-$ and requires that T be real. This breaks the $U(1)_+\otimes U(1)_-$ gauge symmetry to the subgroup preserving $A_+=A_-$ and the reality of T. This subgroup is $U(1)_c\times Z_2$, where $U(1)_c$ is generated by the sum of the generators of $U(1)_+$ and $U(1)_-$, and the Z_2 acts as $T\rightarrow -T$. The Z_2 symmetry of the tachyon potential on a non-BPS D-brane in type II can therefore be viewed as a consequence of a discrete Z_2 gauge symmetry [7].

In the noncommutative case, we similarly begin with the D9$-\overline{\mathrm{D9}}$ system in type IIB and project by $(-1)^{F_L}$ to obtain a non-BPS D9-brane in type IIA. In analogy to the commutative case, the projection by $(-1)^{F_L}$ requires that $A_+=A_-$ and that T be Hermitian. The classical potential for T then has a local maximum at $T=0$, representing the unstable D9-brane and minima at $T=\pm t_*$ representing the closed string vacuum.

In the noncommutative theory, the action is stationary for T in the form

$$T=0P_0+t_*P_+-t_*P_-, \qquad (2.1)$$

where P_0, P_+, and P_- are orthogonal projection operators [1]. Equivalently, we can diagonalize $T=\mathrm{diag}(t_1,t_2,...)$, and the potential is stationary if each t_i is 0, $\pm t_*$. The stable vacua are represented by

$$T_{\mathrm{vac}}=\pm\,\mathrm{diag}(-t_*,-t_*,\dots). \tag{2.2}$$

The discrete Z_2 gauge symmetry discussed above interchanges these two configurations, leaving a single physical vacuum state. k non-BPS D7-branes are represented by

$$T_{\mathrm{D7}}=\pm(0^k,-t_*,-t_*,\dots), \tag{2.3}$$

with the two solutions again interchanged by Z_2. The final solutions of interest are the tensionless 7-branes, given by

$$T_{\mathrm{ten}}=\pm(t_*^k,-t_*,-t_*,\dots). \tag{2.4}$$

We would like to show that T_{ten} is gauge equivalent to T_{vac}, but note that this does not follow from the above Z_2 gauge symmetry.

Instead, we require a gauge symmetry allowing us to flip the eigenvalues of T independently. The need for such a symmetry can be understood on the following grounds: Consider a fluctuation of the tachyon on a single non-BPS D7-brane,

$$T_{\mathrm{D7}}+\delta T_{\mathrm{D7}}=\pm(\delta t,-t_*,-t_*,\dots). \tag{2.5}$$

Since we can construct the non-BPS D7-brane as the $(-)^{F_L}$ projection of the $\mathrm{D7}-\overline{\mathrm{D7}}$ system, it follows that there must be a Z_2 gauge symmetry flipping the sign of the D7 tachyon, $\delta t\to-\delta t$. Generalizing to arbitrary numbers of D7-branes, it follows that there must exist a gauge symmetry allowing us to flip the sign of any collection of eigenvalues, and under this symmetry, T_{ten} is gauge equivalent to T_{vac}. Therefore, the tensionless solutions found in [2,3], with $T=\mathrm{diag}(t_*^k,-t_*,-t_*,...)$, are gauge equivalent to the vacuum and should not be counted as distinct solutions. Similarly, we can show that a superposition of both k non-BPS D7-branes and k' tensionless seven-branes are gauge equivalent to k non-BPS D7-branes. We can always use the Weyl group of $U(\infty)$ to put the tachyon field, Eq. (2.1), into the form $T=\mathrm{diag}(0^{n_0},-t_*^{n_-},t_*^{n_+})$, with $n_0+n_-+n_+$ infinite. For n_0 finite, this solution has the same tension and spectrum as n_0 non-BPS D7-branes [3]. By viewing this solution as a tachyon configuration on n_0+n_- D7-branes, we can use the Z_2 symmetry of the tachyon on these D7-branes to map this solution to the canonical form $T=\mathrm{diag}(0^{n_0},t_*^{n_-},t_*^{n_+})$.

Let us now try to identify this Z_2 gauge symmetry directly. Setting $T=T^\dagger$ and $A_+=A_-$ in the action of the noncommutative $\mathrm{D9}-\overline{\mathrm{D9}}$ system gives

$$S=\int d^8x\,\mathrm{Tr}\left\{D_\mu T D^\mu T-\frac{1}{4}F_{\mu\nu}F^{\mu\nu}-V(T)\right\}, \tag{2.6}$$

with $D_\mu T=\partial_\mu T+i[A_\mu,T]$. Due to the $[A_\mu,T]$ terms, it is clear that for generic A_μ, flipping the sign of a T eigenvalue is not a symmetry of the action—trouble comes from the off-diagonal elements of A_μ.

As discussed in [3], when expanding the action around the background of k D7-branes, the gauge bosons of $U(\infty)/(U(\infty-k)\times U(k))$ appear as unwanted massive degrees of freedom. Their mass cannot be computed reliably in an effective field theory approach, and it was argued that they should be removed by higher derivative terms in the full string field theory. Similarly, we here propose that when expanding around a 7-brane solution, it is necessary to freeze out these off-diagonal degrees of freedom by setting them to zero. Indeed, this is necessary in order to recover the $T\to-T$ gauge symmetry on a non-BPS D7-brane. Of course, it would be desirable to see this happening explicitly, but given this physically well-motivated assumption we have shown that the tensionless solutions are gauge equivalent to the vacuum.

We also note that additional brane and vacuum solutions have appeared in [8,9]. One would like to find a formulation in which the D-brane solutions and vacuum are unique and any additional solutions are gauge artifacts, as we have found here for the tensionless branes in type II theory.

III. A CONSTRUCTION OF D$(2p)$-BRANES

We can also construct nontrivial solutions which interpolate between vacua related by discrete gauge transformations. For example, a BPS D8-brane is represented in the commutative theory by a kink which interpolates between the vacua at $T=\pm t_*$. In the present context we can consider a tachyon field which also depends on one of the commuting directions, say x_7, in which case a D8 (anti-D8)-brane would be given by the configuration

$$T=t_*\Phi_{k,(\bar{k})}(x_7), \tag{3.1}$$

with $\Phi_{k,(\bar{k})}(x_7)$ a kink (antikink) configuration which interpolates between $\mp(\pm)1$ as x_7 varies from $-\infty$ to $+\infty$. Since this configuration depends on a commuting coordinate, it is not possible to compute its tension exactly as in [3]. It is nonetheless clear that it represents a D8-brane (with a B field in two directions along the brane).

We can also construct D6-branes using these ideas in terms of solutions that interpolate between vacua where a finite number of eigenvalues differ in sign [10]. Take $P_\pm$ to be orthogonal projection operators of rank n_+,n_-. Then the solution

$$T=t_*\Phi_k(x_7)P_++t_*\Phi_{\bar{k}}(x_7)P_-+t_*(1-P_+-P_-) \tag{3.2}$$

represents a superposition of n_+ D6-branes and n_- anti-D6-branes. To see this, note that the solution with n_++n_- zeroes on the diagonal, and the remaining diagonal entries equal to t_* represents n_++n_- non-BPS D7-branes. D6-branes are represented by kinks on a non-BPS D7-brane, and the above construction has n_+ kinks and n_- antikinks in commuting subspaces. This construction should be contrasted with the construction of D6-branes as 't Hooft-Polyakov monopoles on several D9-branes in the commutative framework [11]. In Eq. (3.2) only a single D9-brane is required and only the tachyon is excited, whereas in [11] the gauge field is essential.

Similarly, by turning a B field in more directions, one can construct all the BPS D$(2p)$-branes of type IIA as generalized kinks on a single non-BPS D9-brane.

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